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High-spin nuclear states and persistent currents in mesoscopic rings

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Abstract. An analogy is presented between periodic persistent currents in mesoscopic rings and staggerings of gamma energy transitions from some nuclear high-spin states. Various sources of damping of the expected periodic structures in both physical systems are compared. This discussion provides, in the nuclear case, a tentative explanation of the scarcity of such staggerings, their appearance near ^{150}Gd and the existence of a spin-window for their observation.

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1 Introduction

Aharonov and Bohm [1] have revealed some unexpected features of the phase of quantal wavefunctions. They have proven in particular the physical relevance of electromagnetic potentials in quantum mechanics, upon considering the phase shift experienced by a charged particle encircling an infinitely long solenoid (i.e. within a region where the magnetic field is vanishing while the vector potential is not). But apart from that fundamental and spectacular result, they have drawn the physicist's attention on the phase of a wavefunction describing the motion of a charged particle. With a proper use of the boundary conditions satisfied by the wavefunctions, this phase leads to amusing consequences such as in the two following physical systems which will be discussed below.

In recent years, considerable experimental developments have taken place in two totally disconnected domains of microscopic physics. One is opened by the availability of conductor or semi-conductor rings whose diameter is of the order of the micron, obtained by using various advanced techniques (such as electron beam microlithographic techniques or molecular beam epitaxy). At the same time, important progresses have been made in the detection of extremely small variations of the magnetic flux, such as those resulting from the magnetization provoked by an electric current of the order of the nanoampere in such a ring. They take advantage of the development of the SQUID technique (Superconducting Quantum Interference Devices). For the litterature concerning such instrumental developments we refer for instance to [2-3]. It has then been possible [2] to measure persistent currents in an assembly of 10^7 disconnected copper rings of this type when submitted to a magnetic field (of the order of 10 G in [2]) and more recently one has found similar results with a single gold loop using a very high sensitivity SQUID [4]. In another experiment [3] one has developped on the same chip a single GaAs-AlGaAS semiconductor loop and its SQUID measuring device to find qualitatively the same results as in the previous experiments performed with conductor rings. As an important result one has found that the measured currents are periodic with respect to the magnetic flux (with a period related with the elementary quantum of flux $\Phi = h/e$ introduced in [1]). These experimental facts have been searched for as manifestations of the Aharonov-Bohm phase [1].

The second experimental tour de force which we want to mention for our purpose here, has emerged from the development in recent years of gamma 4π detector arrays having both a high intrinsic efficiency and a high granularity (namely at present the EUROBALL and GAMMASPHERE detectors, in Europe and in the USA respectively). The combination of these two characteristics (plus some further instrumental niceties as e.g. anti-Compton filter devices, advanced microelectronical engineering and data handling) have led to very selective detectors able to disentangle rather minute effects out of background (typically capable of sorting out spectroscopic properties associated with some per mil of the total reaction cross section in heavy ion reactions on heavy targets). One of the great achievement of these arrays has been to decipher the response of nuclei to very fast rotations (very fast being understood here as close to the critical angular velocity regime leading to a centrifugal disassembly) allowing then the nucleus to assume very deformed shapes (such as in the so-called superdeformed states). The deexcitation gamma spectra of such states exhibit roughly a linear dependence on the total angular momentum I. This is typical of a rotating charge distribution. Yet, it has been shown recently [5] that in a limited number of superdeformed rotational bands the gamma transition energies exhibit a regular pattern of staggering around a smoothly varying bulk part (i.e. very small deviations, of some tenth of a keV, with an alternate sign above a smooth polynomial behaviour in I). This phenomenon found in some nuclear regions [5-8] has been confirmed in some cases with an independent set of data [8] or infirmed upon redoing the experiment with a better gamma detector array [9]. At any rate, it seems undoubtedly present near and in the ^{149}Gd nucleus for some superdeformed bands [5, 8].

By studying the currents occuring in such rotating systems and out of the close relationship between the currents and the phases of wavefunctions, we propose in this paper to consider this staggering phenomenon as being in a very close analogy with the persistent electric currents measured in mesoscopic rings.

2 Persistent currents in mesoscopic rings

We will schematically discuss here the the physics involved in the mesoscopic ring experiments by making use of a simplified 1D model for the mesoscopic ring, discussing later some properties of a real ring, relevant to our limited purpose. This model introduced by Hund [10] as early as 1938, describes the motion of a free electron moving balistically in the ring (i.e. without collisions with the other particles included in the ring, and in particular with other electrons). This ideal conduction phenomenon has been shown theoretically [11] and experimentally [12] to exist indeed in regimes where the electrons perform essentially elastic scatterings on the impurities of the solid and very few inelastic scattering with the phonons. In such cases, the quantal phase coherence length appears to be order of magnitudes larger than the actual (elastic) mean free path and to be at least of the order of the characteristic length of the studied mesoscopic system. Finally we would like to emphasize that for the analogy which we are going to make, it suffices to analyze the motion of a single electron without entering in the complicated description of how much cooperative effects of several electrons would affect e.g. the period of the persistent current. This allows us to describe qualitatively the quantal electronic motion within the following simplistic approach.

The energy of an electron in a constant magnetic field $\mathbf{B}$ whose vector potential is $\mathbf{A}$, is written

$$E = (\mathbf{p} - e\mathbf{A})^2/2m. \tag{1}$$

For a free electronic motion on a circle of radius R, whose plane is perpendicular to $\mathbf{B}$, the eigenfunctions of the above hamiltonian is defined, in terms of an azimuthal angle θ, as

$$\chi_n(\theta) \propto \exp\left[i\left(k_n R\theta + \int_0^\theta \left(\frac{eA(\varphi)R}{\hbar}\right) d\varphi\right)\right]. \tag{2}$$

The boundary condition over one loop yields the following quantization of the wave number k_n [13]

$$2\pi R k_n + \frac{e\Phi}{\hbar} = 2\pi n, \tag{3}$$

n being an integer number, from where one deduces readily the quantized electron energy

$$E = \frac{\hbar^2}{2mR^2}(n - \Phi/\Phi_0)^2 \tag{4}$$

where Φ is the flux of $\mathbf{B}$ through the surface of the ring. For a given positive value of Φ, the ground state n-value is the integer number closest to $[\Phi/\Phi_0]$ or $[\Phi/\Phi_0]+1$, where $[x]$ stands for the integral part of x. When Φ varies therefore, n raises in steps. The electric current intensity which is equal to $-\partial E/\partial\Phi$ and thus proportional to $n - (\Phi/\Phi_0)$, is a periodic function of Φ with Φ_0 as a period.

3 Coupling a global rotation with a simple intrinsic vortical motion

The well known formal analogy between the equations of motion of charged particles in a stationary uniform magnetic field and of massive particles in a rotating reference frame, suggests that we search now for a possible nuclear analog of the above described physical system. Let us consider high spin nuclear states as experimentally studied currently with the large gamma-ray multidetector arrays presented in Sect. 1. Recently it has been proposed [14] that their collective spectra could be interpreted in terms of the coupling of a global rotation with a uniform intrinsic vortical motion as in Riemann-Chandrasekhar S-type ellipsoids [15]. The corresponding vortical motion may be regarded as a global rotation within the sphere obtained by stretching, upon conserving the volume, the coordinates of an ellipsoid along its principal axes. Just as for the global rotation, this motion is indeed associated with a single angle variable θ and with the quantity $\omega = d\theta/dt$, which we call the angular velocity of the stretched rotation. This motion may be superimposed with the global rotation of an ellipsoid with an arbitrary angular velocity Ω.

Modeling nuclear collective motions in such hydrodynamical terms has a long history (see e.g. the entries (8) to (13) in the reference list of [14]) even though it has not been yet fully studied microscopically with state of the art mean field theories. The latter would correspond to generalized "cranking" approaches stemming from the analogy between canonical point-transformations in classical mechanics and unitary (Thouless) transformations e^{iS} in quantum mechanics, with an operator $\mathbf{S}$ linear in the momentum $\mathbf{p}$ [16]. One would then solve the variational problem

$$\delta\langle H - \boldsymbol{\Omega}.\mathbf{I} - \boldsymbol{\omega}.\mathbf{J}\rangle = 0 \tag{5}$$

where $\mathbf{I}$ is the total angular momentum operator and $\mathbf{J}$ the so-called Kelvin circulation operator (which is simply the orbital angular momentum operator after stretching in both $\mathbf{r}$ and $\mathbf{p}$ [16]). In the particular case of S-type ellipsoids, the intrinsic vortical angular velocity vector $\boldsymbol{\omega}$ is collinear with the angular velocity vector $\boldsymbol{\Omega}$ of the global rotation. The solutions of (5) are thus functions of Ω and ω, the projections of the two angular velocities on their common axis. We assume now that in some (I, J) domain, the nuclear energy can be considered as a quadratic function of Ω and ω

$$E(\Omega, \omega) = \frac{1}{2}A\omega^2 + B\omega\Omega + \frac{1}{2}C\Omega^2 \tag{6}$$

where A, B and C are relevant moments of inertia. It is easy to obtain a hamiltonian operator associated with this energy expression. One first makes use of the standard Hamilton equations

$$\Omega = \partial E/\partial I; \quad \omega = \partial E/\partial J \tag{7}$$

from which one gets

$$I = C\Omega + B\omega; \quad J = B\Omega + A\omega. \tag{8}$$

Upon inverting these equations and inserting the resulting angular velocities as functions of I and J, one obtains an energy which is quadratic in I and J

$$E(I, J) = \frac{1}{AC - B^2}\left(\frac{1}{2}AI^2 - BIJ + \frac{1}{2}CJ^2\right). \tag{9}$$

After some rearrangements of terms, prompted by the description of the motion at given values of I, and after replacing the quantities I and J, canonically conjugated to the angles Θ and θ of which Ω and ω are the time derivatives, by the corresponding operators, one readily obtains

$$\hat{H}(\hat{\mathbf{I}}, \hat{\mathbf{J}}) = \frac{\hat{\mathbf{I}}^2}{2C} + \frac{1}{2}\frac{C}{AC - B^2}\left(\hat{\mathbf{J}} - \frac{B}{C}\hat{\mathbf{I}}\right)^2. \tag{10}$$

Describing the properties of nuclear stationary states which are eigenstates of $\hat{\mathbf{I}}^2$, one may disregard the operator dependence in $\hat{\mathbf{I}}$ of $\hat{H}(\hat{\mathbf{I}}, \hat{\mathbf{J}})$, yielding thus the following equations for the collective intrinsic motion wavefunction $\Phi(I, J)$, corresponding to an eigenstate of the operator $\hat{\mathbf{J}}^2$

$$\begin{aligned} &\frac{1}{2}\frac{C}{AC - B^2}(\hat{\mathbf{J}} - B\vec{\Omega}_{eff})^2\Phi(I, J) = \delta(I, J)\Phi(I, J) \\ &\hat{\mathbf{J}}^2\Phi(I, J) = J(J + \hbar)\Phi(I, J) \end{aligned} \tag{11}$$

where $\Omega_{eff} = I/C$ has the physical meaning of an effective rotational frequency, while $\delta(I, J)$ stands for the energy excess above the classical estimate $E'(I) = I^2/2C$, for the yrast state energy (where as usual in nuclear physics, "yrast state" means the state whose energy is minimal at a given value of I).

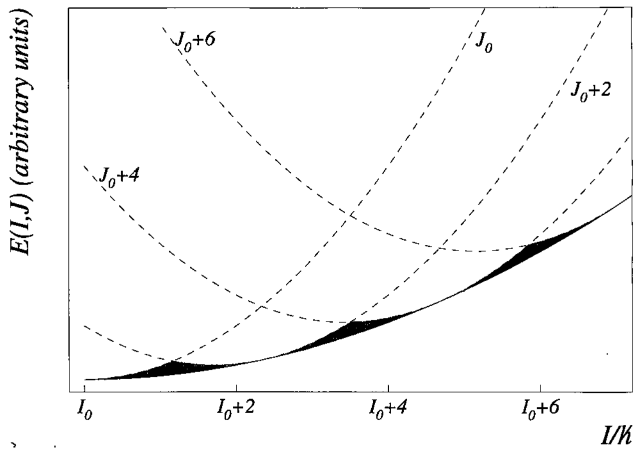

Fig. 1. Illustrative example of collective energies $E(I, J)$, pictured as dashed parabolas, as functions of a continuous variable I for quantized values of J : $J_0\hbar$, $(J_0 + 2)\hbar$, ... , where J_0 is an even non-negative integer number. The parabolic envelope of these energies, corresponding to the classical yrast energy, is also drawn. The periodic (in I) difference between the quantal and the classical yrast lines, $\delta(I, J)$, is represented as the *black area* on the Figure

In the above also, "classical" means that we have considered J as a continuous variable. In fact, it is important to note that $\mathbf{J}$ is quantized as an orbital angular momentum. This property is easily demonstrated by calculating the commutator between two components of $\mathbf{J}$. It is also a direct consequence of the matching property after one loop of the wavefunction depending on θ which is canonically conjugated with $\mathbf{J}$. This is a one-to-one transcription of the boundary condition problem yielding (3). It is clear that such a quantization of $\mathbf{J}^2$ makes our approach crucially different from the one of Rosensteel [17] even though our definition of the operator $\mathbf{J}$ is obviously identical. On the other hand, we have assumed here that we are in a "fast rotation" regime (i.e. I and $J \gg \hbar$ so that we have replaced e.g. $J(J + \hbar)$ by J^2), noting that releasing this restriction would slightly complicate the mathematical expressions without altering the main conclusions (see the discussion of [16]).

The parabolic behaviour of the energy as a function e.g. of I for a given quantized value of J, the envelope of such curves and the periodic (in I) difference between the latter and the quantized (in J) yrast energy are illustrated on Fig. 1.

4 Analogy between idealized currents in mesoscopic rings and gamma transitions between states belonging to a super-deformed rotational band

The analogy between the physical systems described in the two previous sections may thus be detailed as follows. The nuclear global rotation plays the role of the constant magnetic field while the intrinsic vortical mode corresponds to the circular displacement of the point charge. In both cases the motions are described by a single angular variable θ.

The first of the equations (11) establishes formally the above analogy between the motion of an electron in a permanent magnetic field $\mathbf{B}$, whose wavefunction is an eigenstate of the projection on the $\mathbf{B}$-direction of the orbital momentum $\boldsymbol{l}$, on one hand with the intrinsic vortical motion of some nuclear matter enclosed in a rotating container, whose wavefunction is an eigenstate of $\mathbf{I}^2$ and $\mathbf{J}^2$, on the other hand. Now, $\mathbf{I}$, or a quantity proportional to it having the dimensions of an angular velocity called here $\boldsymbol{\Omega}_{\mathit{eff}}$, will be associated with a quantity proportional to a uniform field $\mathbf{B}$, while $\mathbf{J}$ will be associated with $\boldsymbol{l}$. The former analogy ($\boldsymbol{\Omega}_{\mathit{eff}} \propto \mathbf{B}$) is rather well-known when describing the rotation of a charged point particle, or the global rotation of a fluid, i.e. without intrinsic vortical motion. The inertia parameter B, generating the coupling of the two collective modes of the fluid, plays the role of the charge of the particle coupling its motion to the magnetic field. At a given value of I the nuclear energy $\delta(I,J)$ defined in the first (11), may be written as

$$\delta(I,J) = \frac{\hbar^2 C}{2(AC - B^2)}(j - \Psi/\Psi_0)^2 \tag{12}$$

with $j = \frac{J}{\hbar}$; $\Psi = \frac{B}{C}I$; $\Psi_0 = \hbar$.

Such an expression is manifestly of the same nature as what has been obtained for the mesoscopic ring in (4). The analog of the ground state in this case is the yrast state. Its energy $\delta(I,J)$, relative to the base line $E'(I)$, presents the property of being raised by steps due to the quantized character of J, similar to what has been found for the electron. It may be noted that in the mesoscopic ring case, the quantized energy of (4) can also be represented by the drawing made in Fig. 1, where the abscissa represents now the magnetic flux and upon replacing the parabolic envelope by the zero energy horizontal line.

The analog of the electric current intensity is the derivative of with respect to I or equivalently to the "flux" Ψ. It represents in the nuclear case, the fluctuating part of the gamma transition energy between two yrast states having neighbouring values of I. As does the electric current in mesoscopic rings, it will exhibit periodic jumps as a function of I, resulting thus in a staggering of these transition energies. This is indeed observed in the data pertaining to the gamma deexcitation of some superdeformed nuclear states in ^{149}Gd [5], ^{148}Gd and ^{148}Eu [8] and possibly as well in the ^{132}Ce region [6]. Its interpretation as resulting from the quantization of an intrinsic vortical mode has been already tentatively proposed in reference 14. Alternative descriptions have been proposed (C_4 symmetry [18], collective kinetic energy with fourth order terms in the angular momentum [19], band mixing within a projected shell model approach [20] or within some versions of the Interacting Boson Model [21, 22]). In the former case however, it does not seem to be grounded on any microscopic calculations for the hexadecapole deformation [23]. In turn, the difficulty met, in particular, by our tentative explanation [14], is its seemingly very general character which does not fit with the scarcity of such a staggering. Moreover, one has also to provide a rationale for the somewhat limited "spin window" in which this phenomenon takes place. The analogy sketched here may be useful to understand these two facts.

5 Discussion

Let us release some among the above performed simplifications for both physical systems. We will proceed by considering first the ring physics and then transpose the discussion to the context of its nuclear analog. As a first point of discussion, we note that actual rings have indeed a finite cross-section. This finiteness entails a distribution of the ring radius R and thus of the flux Φ. The existence of periodic persistent currents would still prevail, provided that the fluctuation ΔR of R is small with respect to its average value $\bar{R}$, to which corresponds an average flux $\bar{\Phi}$, namely

$$2\bar{\Phi}\Delta R/\bar{R} \ll \Phi_0. \tag{13}$$

In the nuclear case, the role of R is played by the ratio $x = B/C$ which is mostly deformation dependent. An equivalent to the distribution in R is the spread in nuclear shapes resulting from the coupling of nuclear rotations and vibrations. To estimate the importance of this effect, we will model the superdeformed nucleus as an axially symmetrical ellipsoid of axis ratio q. From the semiclassical estimate of [14] for B and C, one obtains $x = 2/(q+1/q)$, to the lowest order in $\hbar$. The condition (13) for the existence of periodic currents in closed rings would then become in the nuclear case

$$\frac{|\Delta q|}{q} \ll q\frac{\hbar}{2I}\,\frac{(1+1/q^2)^2}{(1-1/q^2)} \tag{14}$$

For given values of q and of its fluctuation Δq, the validity of such a condition if met at some value of the spin I, would disappear upon sufficiently increasing I. On the other hand, it appears that Δq increases when decreasing I (due to the disappearance of the inner barrier in the rare-earth region whose existence implies a sufficient angular velocity, or to an increase of the coupling to normally deformed states in the Hg region [24]). Both limits might therefore explain the observed "spin-window" for the staggering phenomenon.

It is worth noting now that the above condition for the existence of a staggering should be only marginally met in most nuclei. For instance, assuming that $I = 30\hbar$ and $q = 1.8$, one requires that Δq should be much smaller than 0.13, which is not very large indeed as compared with the calculated widths of vibrational intrinsic states for superdeformed states in some Hg [25] or Gd [26] isotopes. In the latter case, for instance, one has estimated in reference 26, the quadrupole moment fluctuation ($\sqrt{\langle Q^2\rangle - \langle Q\rangle^2}$) in the vicinity of ^{150}Gd to generally correspond to a fluctuation of the parameter q of the order of precisely 0.13 (for $q \sim 1.8$ and $A \sim 150$).

Clearly the estimate of (14) is purely semiclassical, it indicates nevertheless that in most cases, the condition for a staggering should not be met. The actual existence of a staggering mode should thus correspond to a significant local deviation from the average values of Δq due to structural effects originating, for instance, from a sudden increase of the axial quadrupole mass parameter $M(Q)$ in the relevant deformation range. As a matter of fact, recent perturbative calculations of $M(Q)$ have been performed [26] for the ^{146,148,150,152,154}Gd isotopes. Whereas they show no significant structure in the superdeformed region in all cases but for ^{150}Gd, they yield for the latter a dramatic increase of $M(Q)$ (by a factor larger than 50 near the maximum) exactly in phase with the local minimum. This yields a lowering of the quadrupole moment fluctuation by a factor of three roughly, allowing thus the condition of (14) to be almost met. It is furthermore argued in [26] that, contrarily to the other isotopes studied there, upon increasing the angular momentum in ^{148}Gd, one retrieves a situation which is very much comparable with what we have discussed for ^{149}Gd at zero angular momentum. This might provide a qualitative explanation for the staggerings observed in ^{149}Gd, ^{148}Gd and ^{148}Eu.

In the mesoscopic ring physics, it is known that the existence of transverse modes, allowed by the 3D character of a real ring, results also in a damping of the periodic current. As a matter of fact, one must not consider a single circular orbit but a bunch of them corresponding to different quantal states for the transverse electron energy. In the high-spin limit of the nuclear case, the 3D nature of rotations is expected to generate a wobbling-like motion [27]. The model hamiltonian must be modified so as to incorporate it. In [28], we have generalized the quantal treatment of the precession motion described in [27] for the case of global rotations alone, in the limit where the **I** and **J** momenta have one projection on a principal axis which is much larger than the others. In this case one creates for each value of the quantum numbers I and J, a spectrum of boson excitations. It has also been noted in [28] that the zero-point wobbling motion does renormalize the energy resulting thus in a modification of the parameters A, B and C in the quadratic expression of the energy (9). Incidentally the latter effect may also yield a substantial deviation from the quadratic character of the collective energy itself which has been assumed here. This simplification corresponds indeed to an assumption on the constancy of the inertia tensor with respect to the total angular momentum, which is roughly substantiated by experimental data in most SD bands but in no case exactly fulfilled either.

The description of the two physical systems which we have made here, is of a purely kinetic nature. A more complete description of them would require to add to the collective kinetic energy, terms representing the interaction of this mode with the rest of the system. In the mesoscopic ring case one could add a potential $V(\theta)$ to take into account approximately the elastic collisions of the electrons with the solid impurities. A treatment of the inelastic scattering on the phonons of the solid would be more complicated.

Similarly in the nuclear case, we have neglected in particular the coupling with other collective modes, excepted the global rotation. As a matter of fact, our description of the intrinsic vortical mode conserves the Kelvin circulation **J**. Such an approximation depicts a frictionless nuclear matter flow which conserves the current. This could be improved, for instance, by introducing a complex potential in the hamiltonian of (10) whose imaginary part would describe the energy transfer from the studied collective mode to the others, in cases where a full quantal treatment would prove to be beyond reach. Taking furthermore into account the couplings with non-collective modes, one could imply the concept of nuclear viscosity whose assessment, however, is a rather difficult task and is obviously contingent upon the considered process. Intrinsic vortical modes being essentially shape-conserving, should not yield much of one-body viscosity, which is mostly generated in other processes by the moving nuclear surface. However it is impossible to rule out some residual viscosity effect, in particular of two-body character, providing thus a further damping of the discussed staggering phenomenon. Moreover in [14], a manifestation of the J-mixing has been advocated to explain the observed intraband transition within e.g. the yrast band. Even though the experimental data cannot specify the exact amount of such a mixing, however they clearly prove its existence.

6 Conclusions

A possible analogy between two seemingly very different physical systems has been established. The discussion of the physics of the mesoscopic ring has been only slightly touched here. Clearly we use this analogy to better understand our recently proposed description of very high spin in terms of a coupling of global rotations with intrinsic vortical modes. While this approach seemed quite general in nature, it remained to explain why the gamma transition energy staggering which may be deduced from it, was so rarely found in relevant nuclear spectroscopic data. The somewhat elusive character of this alleged manifestation of the nuclear intrinsic vortical motion is mirrored by the experimental difficulty of exhibiting persistent currents in mesoscopic rings. A major result of our approach, is the qualitatively explanation of this experimental scarcity, without ruling out the possibility of its accidental occurence (as near ^{150}Gd), as resulting from various damping mechanisms (stemming from a deformation distribution, quantal wobbling modes and the two-body viscosity). Furthermore a tentative explanation for the existence of a “spin-window” for the staggering pattern has been proposed. It is clear that the general arguments presented here, call for more specific microscopic estimates. In general such intrinsic vortical currents are sufficiently damped, so that they do not manifest themselves through the considered staggering. This does not preclude, a priori, the widely spread character of their existence. On the contrary, demonstrating their relevance through such a scarce phenomenon might be the telltale indicator of their general presence. However, it remains still to qualitatively explain within our framework why such staggerings are present in some superdeformed bands and not in others within the same nucleus (possibly through the explicit coupling of the intrinsic vortical modes with the quasiparticle degrees of freedom).

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